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The fundamental optimal relations of the allocation, cost and effectiveness of the heat exchangers of a Carnot-like power plant

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Abstract
A stationary Carnot-like power plant model, with three sources of irreversibilities (the finite rate of heat transfers, heat leak and internal dissipations of the working fluid), is analyzed by a criterion of partial optimization for five objective functions (power, efficiency, ecological function, efficient power and $\dot{\Omega}$ criterion). A remarkable result is that if two constraints (design rules) are applied alternatively: constrained internal thermal conductance or fixed total area of the heat exchangers from hot and cold sides; the optimal allocation, cost and effectiveness of the heat exchangers are the same for all these objective functions independently of the transfer heat law used. Thus, it is enough to find these optimal relations for only one, maximum power, when all heat transfers are linear. In particular, for the Curzon–Albhorn-like model (without heat leak), the criterion for the so-called ecological function, including other variables (the internal isentropic temperature ratio), becomes total.

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1. Introduction

Recently in the conclusions of [1], a criterion of partial optimization for four objective functions (power, efficiency, the so-called ecological function and entropy generation [2]) has been presented, and in which the optimal characteristic parameters were: the allocation, cost and effectiveness of the heat exchangers of an irreversible Carnot cycle with linear finite rate heat transfers between the working fluid and its two heat reservoirs, and linear heat leak. Although the criterion of partial optimization is easily applicable to any cyclic model, a standard irreversible Carnot-like cycle was chosen because of its simplicity to account for the main irreversibilities that usually arise in real heat engines: finite rate heat transfer between the working fluid and the external heat sources, internal dissipation of the working fluid and heat leak between reservoirs [3]. This cyclic model was originally proposed by Chen [4] and simultaneously by Yan [5] with different approaches for the internal dissipation (see also Gordon [6]). Formerly, Bejan [7–10] considered this model but without internal dissipation and Curzon–Alhborn [11] when there is not heat leak in the model, i.e. the only source of irreversibility in the engine (CA-engine) is a linear finite rate heat transfer between the working fluid and its two heat reservoirs [12].

The above Carnot-like cyclic model has been studied at length for many objective functions, besides power, efficiency, ecological function and entropy generation, different transfer heat laws and several characteristic parameters.

The maximum power and efficiency have been obtained in [4, 5, 13]. The maximum ecological function was obtained in [14] for the CA-engine and in a more general form in [15–17]. Bejan [10] has considered the minimization of entropy generation. In general, these optimizations were performed with respect to only one characteristic parameter: the internal isentropic temperature ratio. In the first analysis of the CA-engine, the time ratio of heat transfer from the hot to the cold side was considered, but in further works this ratio was not taken into account (see the reviews of [18–21] for more details). In [1, 22], this ratio was taken into account as another characteristic parameter of the engine, and we found that the time allocation of heat transfer between the hot and cold sides is the same for maximum power and efficiency, and it is also the same for the maximum ecological function and minimum entropy generation [1]. In this same work, the optimization with respect to other parameters, such as the allocation ratio of the heat exchangers [8] and the cost and effectiveness ratio of the heat exchangers [2, 23], for this Carnot-like cyclic model was performed. As a consequence, the partial criterion mentioned above was obtained.

On the other hand, effects of heat transfer laws or when a property is independent of the heat transfer law for this Carnot-like cyclic model have been discussed in several works [16–20, 24–29], and so on (see also references included there). Moreover, the optimization of other objective functions (efficient power, $\dot{\Omega}$ criterion, and so on) has been analyzed [3, 18–21, 30–32].

In what follows, a stationary Carnot-like power plant model (see figure 1), with irreversibilities of the finite rate of heat transfers between the heat engine and its reservoirs, heat leak between the reservoirs and internal dissipations of the working fluid, is analyzed by a criterion of partial optimization for several objective functions (power, efficiency, ecological function, efficient power and $\dot{\Omega}$ criterion). We have found that optimal allocation, cost and effectiveness of the heat exchangers are the same for all these objective functions. Besides these optimal values are invariant to the law of heat transfer used, including the heat leak, if two constraints are applied: constrained internal thermal conductance or fixed total area of the heat exchangers from the hot and cold sides.

This paper is organized as follows. In section 2, the stationary Carnot-like power plant model and a criterion of partial optimization are presented. In section 3, the optimal expressions for allocation, cost and effectiveness of the heat exchangers corresponding to maximum power are shown when all the heat transfers are assumed to be linear in temperature differences. Section 4 is devoted to discussions and conclusions.

2. The Carnot-like power plant and a criterion of partial optimization

The class of the Carnot-like power plant model shown in figure 1 satisfies the following conditions for

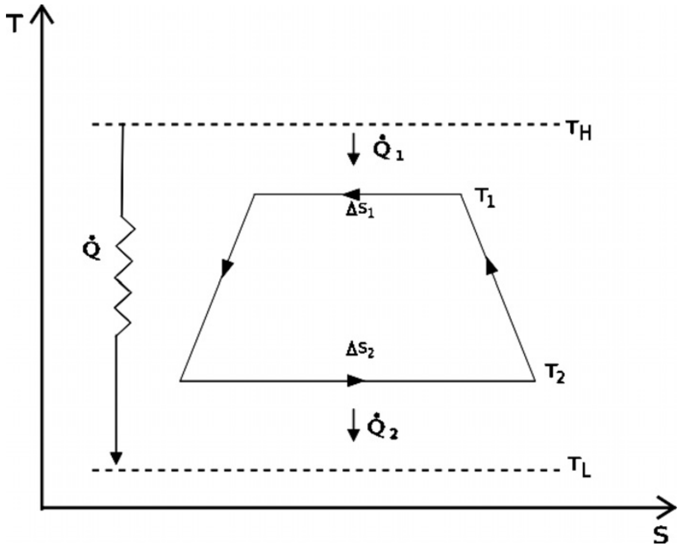

Figure 1. A Carnot-like power plant with heat leak and finite heat transfer rates, and internal dissipations of the working fluid.

(i) The working fluid flows through the system in a stationary state. The cycle of the power plant consists of two isothermal and two adiabatic processes. The temperatures of the working fluid in the hot and cold isothermal processes are, respectively, T_1 and T_2. The temperatures of the hot and cold heat reservoirs are, respectively, T_H and T_L.

(ii) There is thermal resistance between the working fluid and heat reservoirs.

(iii) There is a heat leak rate from the hot reservoir to the cold reservoir [7]. In real-power plants leaks are unavoidable. There are many features of an actual power plant which fall under that kind of irreversibility, such as the heat lost through the walls of a boiler, a combustion chamber, or a heat exchanger, and heat flow through the cylinder walls of an internal combustion engine, and so on.

(iv) Besides thermal resistance and heat leak, there are other irreversibilities in the power plant: the internal irreversibilities. For many devices, such as gas turbines, automotive engines and thermoelectric generator, there are other loss mechanisms, i.e. friction or generators losses, and so on, that play an important role, but are hard to model in detail. Some authors use the compressor (pump) and turbine isentropic efficiencies to model the internal loss in the gas turbines or steam plants [33]. Others, in Carnot-like models, use simply one parameter to describe the internal losses. This parameter is associated with the entropy produced inside the power plant during a cycle and makes the Claussius inequality an equality (for details see [18]):

$$\frac{\dot{Q}_2}{T_2} - I\frac{\dot{Q}_1}{T_1} = 0, \tag{1}$$

where $\dot{Q}_i (i = 1, 2)$ are the heat transfer rates and $I = \frac{\Delta S_2}{\Delta S_1} \geqslant 1$ [4].

The heat transfer rates $\dot{Q}_H$, $\dot{Q}_L$ transferred from the hot–cold reservoirs are given by

$$\dot{Q}_H = \dot{Q}_1 + \dot{Q} \tag{2}$$

$$\dot{Q}_L = \dot{Q}_2 + \dot{Q}, \tag{3}$$

where the heat leak rate is $\dot{Q}$ which is positive, and $\dot{Q}_1$, $\dot{Q}_2$ are the finite heat transfer rates between the reservoirs T_H, T_L and the working substance of the model.

By first law and combining equations (1) and (2), the power P and heat transfer rate $\dot{Q}_H$ are given by

$$P = \dot{Q}_H - \dot{Q}_L = \dot{Q}_1 - \dot{Q}_2 = \dot{Q}_1(1 - Ix) \tag{4}$$

$$\dot{Q}_H = \dot{Q}_1 + \dot{Q} = \frac{P}{1 - Ix} + \dot{Q}, \tag{5}$$

where $x = \frac{T_2}{T_1}$ is the internal isentropic temperature ratio.

The thermal efficiency is given by

$$\eta = \frac{P}{f(x)P + \dot{Q}}, \tag{6}$$

where $f(x) = \frac{1}{1-Ix}$ is positive.

The entropy-generation rate is (see the comment of Yan in [16])

$$S_{\text{gen}} = \frac{\dot{Q}_L}{T_L} - \frac{\dot{Q}_H}{T_H} > 0.$$

Then, the entropy-generation rate multiplied by the temperature of the cold side gives us a function Σ, which is (equations (2) and (4))

$$\Sigma = T_L S_{\text{gen}} = T_L \left(\frac{\dot{Q}_H - P}{T_L} - \frac{\dot{Q}_H}{T_H} \right) = \dot{Q}_H(1 - \mu) - P$$

so,

$$\Sigma = g(x)P + \dot{Q}(1 - \mu), \tag{7}$$

where $\mu = \frac{T_L}{T_H}$ and $g(x) = f(x)(xI - \mu)$ is also positive, since $1 - Ix$ must be less than the Carnot efficiency $1 - \mu$.

The ecological function [14], if T_L is considered as the environmental temperature, is given by

$$E = P - \Sigma$$
$$E = (1 - g(x))P - \dot{Q}(1 - \mu), \tag{8}$$

and $g(x)$ is less than 1, since, if there is no heat leak, the following inequality is satisfied ([15, 16] and comments of Yan there):

$$1 < \frac{P}{\Sigma} = \frac{1}{g(x)}.$$

The efficient power is defined as power times efficiency [31] (cf with [34]):

$$P_\eta = \eta P. \tag{9}$$

Finally, the $\dot{\Omega}$ criterion [30] states a compromise between energy benefits and losses for a specific job and for the Carnot-like power plant discussed herein, it is expressed as [3]

$$\dot{\Omega} = \frac{2\eta - \eta_{\max}}{\eta} P, \tag{10}$$

where $\eta_{\max}$ can assume to be a constant.

The six objective functions above ($P, \eta, \Sigma, E, P_\eta$ and $\dot{\Omega}$) will depend only on two variables: x (the isentropic temperature ratio) and ϕ which will be related to two alternate constraints (design rules [35]) for the heat exchange at the two ends of the Carnot-like power plant model. It also relates to the heat transfer rate cost, which differs from the hot and cold sides of the model [23].

The first design rule corresponds to the allocation of the heat exchangers [9]. The thermal conductances α, β of the hot and cold sides, respectively, can be written as

$$\alpha = UA_H; \qquad \beta = UA_L,$$

where U is the overall heat transfer coefficient, and A_H and A_L are the available areas for heat transfer. Then, a first approach is to suppose that U is fixed, in both the hot side and the cold side heat exchangers, and that the area A can be allocated between both. The optimization problem is to select the best allocation ratio. To take UA as a fixed value can be justified in terms of the area purchased, and the fixed running and capital costs that altogether determine the overall heat transfer coefficient [23].

Thus, the first design rule is given by the constrained internal thermal conductance of the Carnot-like power plant model:

$$\alpha + \beta = \gamma, \tag{11}$$

where γ is a constant, which is applied to the allocation of the heat exchangers from the hot and the cold side with the same overall heat transfer coefficient U by unit of area A in both ends. Then,

$$\frac{\alpha}{U} + \frac{\beta}{U} = A,$$

and parametrize it as

$$\phi = \frac{\alpha}{UA}; \qquad 1 - \phi = \frac{\beta}{UA}. \tag{12}$$

Also, ϕ can correspond to the cost of providing the heat transfer rate which differs for the hot reservoir and the cold reservoir of the power plant. Let this be represented as having a cost per unit heat transfer: a on the hot side and b on the cold side. Then,

$$a\alpha + b\beta = C, \tag{13}$$

where C is the fixed total cost. Thus, we have that the characteristic parameter above changes to

$$\phi = c\alpha; \qquad 1 - \phi = c\beta, \tag{14}$$

where $c = \frac{a}{C}$.

Alternatively, we may face an existing heat exchange apparatus which is to be redistributed between the hot and cold sides to achieve optimum operation regimes. Now, the total area A is fixed but when distributed it has different overall heat transfer coefficients and hence different effectiveness on the hot and cold sides [1]. This is the second design rule which corresponds to

$$A = A_H + A_L = \frac{\alpha}{U_H} + \frac{\beta}{U_L}, \tag{15}$$

where A_H, A_L are heat transfer areas and U_H, U_L are overall heat transfer coefficients on the hot and cold sides, respectively.

In parametrizing again:

$$\phi = \frac{\alpha}{U_H A}; \qquad 1 - \phi = \frac{\beta}{U_L A}. \tag{16}$$

The optimization of five objective functions (P, η, E, P_η and $\dot{\Omega}$) for the parameter x is well known, including different heat transfer laws (maximum power, efficiency or ecological function ([1, 18–21], and references included therein), and maximum efficient power [31] and $\dot{\Omega}$ criterion [3]). The remaining objective function Σ ($T_L S_{\text{gen}}$) does not have a minimum for the variable x, within the valid interval, as was shown in [1] and for the variable ϕ, neither has Σ a minimum within its valid interval, as will be shown in what follows. Henceforth, x will be fixed and we will assume that the law of heat transfer can be any law, including also the heat leak, with internal thermal conductances, overall heat transfer coefficients, areas and costs given by equations (11), (13) or (15).

Now, if ϕ_{mp} is the point in which the power P achieves a maximum value, then

$$\left.\frac{\partial P}{\partial \phi}\right|_{\phi_{mp}} = 0 \qquad \text{and} \qquad \left.\frac{\partial^2 P}{\partial \phi^2}\right|_{\phi_{mp}} < 0.$$

We will apply it to each one of the other objective functions.

First, the power and the efficiency satisfy the functional relationship given by equation (6). As

$$\frac{\partial \eta}{\partial \phi} = \frac{\dot{Q}\left(\frac{\partial P}{\partial \phi}\right)}{[f(x)P + \dot{Q}]^2}$$

since $\dot{Q}$ does not depend on the variable ϕ. Therefore,

$$\left.\frac{\partial \eta}{\partial \phi}\right|_{\phi_{me}} = 0 = \left.\frac{\partial P}{\partial \phi}\right|_{\phi_{mp}},$$

where ϕ_{me} is the point in which the efficiency η achieves a maximum value.

This implies that their roots are the same $\phi_{mp} = \phi_{me}$ (necessary condition). Also, it is easily seen that for $\phi_{mp} = \phi_{me}$, the power and the efficiency reach a maximum (sufficiency condition), since

$$\left.\frac{\partial^2 \eta}{\partial \phi^2}\right|_{\phi_{mp}=\phi_{me}} = \frac{\dot{Q}\left(\left.\frac{\partial^2 P}{\partial \phi^2}\right|_{\phi_{mp}=\phi_{me}}\right)}{[f(x)P + \dot{Q}]^2} < 0.$$

Second, the power and the entropy generation satisfy the functional relationship given by equation (7). Now, as

$$\left.\frac{\partial \Sigma}{\partial \phi}\right|_{\phi_{m\Sigma}} = 0 = \left.\frac{\partial \Sigma}{\partial \phi}\right|_{\phi_{mp}} \qquad \text{and} \qquad \left.\frac{\partial^2 \Sigma}{\partial \phi^2}\right|_{\phi_{mp}=\phi_{m\Sigma}} = g(x)\left(\left.\frac{\partial^2 P}{\partial \phi^2}\right|_{\phi_{mp}=\phi_{m\Sigma}}\right) < 0$$

then, for this model, the entropy generation does not have a minimum because the sufficiency condition is not satisfied. For the variable x, the entropy generation does not have a minimum within its valid interval [1]. Therefore, this objective function does not have a global minimum.

Third, the power and the ecological function E satisfy the functional relationship given by equation (8). Then,

$$\left.\frac{\partial E}{\partial \phi}\right|_{\phi_{mec}} = \left.\frac{\partial P}{\partial \phi}\right|_{\phi_{mp}} - \left.\frac{\partial \Sigma}{\partial \phi}\right|_{\phi_{m\Sigma}} = 0 \quad \text{and} \quad \left.\frac{\partial^2 E}{\partial \phi^2}\right|_{\phi_{mp}=\phi_{mec}} = (1 - g(x))\left(\left.\frac{\partial^2 P}{\partial \phi^2}\right|_{\phi_{mp}=\phi_{mec}}\right) < 0,$$

where ϕ_{mec} is the point in which the function E achieves a maximum value.

Fourth, the power and the function P_η satisfy the functional relationship given by equation (9). As

$$\frac{\partial P_\eta}{\partial \phi} = P\frac{\partial \eta}{\partial \phi} + \eta\frac{\partial P}{\partial \phi}$$

so,

$$\left.\frac{\partial P_\eta}{\partial \phi}\right|_{\phi_{pme}} = 0 = \left.\frac{\partial P}{\partial \phi}\right|_{\phi_{mp}} = \left.\frac{\partial \eta}{\partial \phi}\right|_{\phi_{me}}.$$

It is easily seen that for $\phi_{mp} = \phi_{me} = \phi_{pme}$, the power, efficiency and P_η reach a maximum (sufficiency condition) since,

$$\left.\frac{\partial^2 P_\eta}{\partial\phi^2}\right|_{\phi_{mp}=\phi_{me}=\phi_{pme}} = \left.\left(P\frac{\partial^2\eta}{\partial\phi^2}\right)\right|_{\phi_{mp}=\phi_{me}=\phi_{pme}} + \left.\left(\eta\frac{\partial^2 P}{\partial\phi^2}\right)\right|_{\phi_{mp}=\phi_{me}=\phi_{pme}} < 0$$

because $\left(\eta\frac{\partial P}{\partial\phi}\right)\big|_{\phi_{mp}=\phi_{me}=\phi_{pme}} = 0$ and since P and η are positives in $\phi_{mp} = \phi_{me} = \phi_{pme}$.

Fifth, the power and the $\dot{\Omega}$ function satisfy the functional relationship given by equation (10) with $\dot{Q} > 0$. As

$$\left.\frac{\partial\dot{\Omega}}{\partial\phi}\right|_{\phi_{mp}=\phi_{me}=\phi_{m\Omega}} = 2\left(\left.\frac{\partial P}{\partial\phi}\right|_{\phi_{mp}=\phi_{me}=\phi_{m\Omega}}\right) + \eta_{\max}\left(\left.\left(\frac{P}{\eta^2}\frac{\partial\eta}{\partial\phi}\right)\right|_{\phi_{mp}=\phi_{me}=\phi_{m\Omega}} - \left.\left(\frac{1}{\eta}\frac{\partial P}{\partial\phi}\right)\right|_{\phi_{mp}=\phi_{me}=\phi_{m\Omega}}\right) = 0$$

Moreover,

$$\left.\frac{\partial^2\dot{\Omega}}{\partial\phi^2}\right|_{\phi_{mp}=\phi_{me}=\phi_{m\Omega}} = \left.\left(\frac{2\eta-\eta_{\max}}{\eta}\frac{\partial^2 P}{\partial\phi^2}\right)\right|_{\phi_{mp}=\phi_{me}=\phi_{m\Omega}} + \eta_{\max}\left.\left(\frac{P}{\eta^2}\frac{\partial^2\eta}{\partial\phi^2}\right)\right|_{\phi_{mp}=\phi_{me}=\phi_{m\Omega}} < 0$$

since $\dot{\Omega}$ is positive in $\phi_{m\Omega}$ and

$$\left.\frac{\partial P}{\partial\phi}\right|_{\phi_{mp}} = \left.\frac{\partial\eta}{\partial\phi}\right|_{\phi_{me}} = 0.$$

Thus, we have obtained

$$\phi_{mp} = \phi_{me} = \phi_{mec} = \phi_{pme} = \phi_{m\Omega} \tag{17}$$

independently of the transfer heat law used.

Therefore, we have the following criterion of partial optimization.

Criterion 1. *Let $F(x,\phi) = \eta(x,\phi), E(x,\phi), P_\eta(x,\phi), \dot{\Omega}(x,\phi)$ (efficiency, ecological function, efficient power or $\dot{\Omega}$ criterion, respectively) where x corresponds to the internal isentropic temperature ratio and ϕ is a variable that corresponds to the allocation, cost per unit heat transfer or different effectiveness of the heat exchangers for the class of stationary Carnot-like power plant models analyzed. If ϕ_{mp} is the point in which the power P achieves a maximum value then ϕ_{mp} is the point in which the function $F(x,\phi)$ achieves a maximum value (equation (17)). This optimization is valid for any law of heat transfer that satisfies at least one of the equations (11), (13) or (15).*

A remarkable conclusion of this criterion is that it can find the maximum for one and only one objective function, for example the power, for the Carnot-like power plant model, by

$$\left.\frac{\partial P}{\partial\phi}\right|_{\phi_{mp}} = 0 \qquad \text{and} \qquad \left.\frac{\partial^2 P}{\partial\phi^2}\right|_{\phi_{mp}} < 0,$$

where ϕ corresponds to one of the variables mentioned in the criterion above. After the value of equation (17) is substituted in the appropriate functional relationships (equations (6)–(10)), the obtained $F\left(x,\phi_{mp}\right) = F(x)$ can be, then, optimized with respect to the x parameter only as was performed for the efficiency in [13]. Thus, we have found that the partial criterion optimizes the other objective functions. Moreover, this optimization is a property independent from the heat transfer law.

Finally, the criterion yields that it is enough to have one, and only one, algebraic expression of one of the objective functions and any heat transfer law. Thus, we can choose for our convenience the power and the conduction heat transfer law since they are algebraically the simplest.

3. Optimum ϕ for the dimensionless power output

If all heat transfer rates are assumed to be linear in temperature differences, the dimensionless power output, $p = \frac{P}{UAT_H}$, if ϕ is given by equation (12) (the first rule (11)), is [1]

$$p = \frac{(1-\sqrt{I\mu})^2}{\frac{1}{\phi} + \frac{I}{1-\phi}} \tag{18}$$

because x_{mp} is the point in which the power P achieves a maximum value and it is given by [13]

$$x_{mp} = \sqrt{\frac{\mu}{I}}. \tag{19}$$

Now, if ϕ_{mp} is the point in which the power P reaches its maximum, ϕ_{mp} is given by

$$\phi_{mp} = \frac{1}{\sqrt{I}+1}. \tag{20}$$

Then, it follows from equation (20) that when the power plant operates at maximum power, also for the optimization of the other objective functions given by the criterion of partial optimization, the relation of the heat transfer areas from the cold side to the hot side is

$$\begin{array}{ll} \frac{A_L}{A_H} = \sqrt{I} \geqslant 1; & \frac{\beta}{\alpha} = \frac{UA_L}{UA_H} = \sqrt{I} \\ A_L = \sqrt{I}A_H & \end{array}. \tag{21}$$

This result shows that the size of the heat exchanger in the cold side must be larger than the size of the heat exchanger in the hot side. In accordance with the definitions adopted by the internal conductance, if $I > 1$, the one for the cold side results greater than the one for the hot side.

If, instead of equation (12), equation (14) is applied, the dimensionless power, $p^* = \frac{P}{C^*T_H}$ with $C^* = \frac{C}{a}$, is given by

$$p^* = \frac{(1-\sqrt{I\mu})^2}{\frac{1}{\phi^*} + \frac{cI}{1-\phi^*}},$$

where $c = \frac{b}{a} > 1$ because of equation (21).

In optimizing the power with respect to ϕ^*, we have

$$\phi^*_{mp} = \frac{1}{1+\sqrt{cI}} \tag{22}$$

or equivalently

$$\frac{\beta}{\alpha} = \sqrt{\frac{I}{c}}. \tag{23}$$

Of course, this reverts to the earlier form (equation (21)) if $c = 1$.

In order to apply the second rule (15), we may face an existing heat exchange rate apparatus which is to be redistributed between the hot and cold sides to achieve maximum power. Now, the total area A is fixed but when distributed it has different overall heat transfer coefficients and hence different effectiveness on the hot and cold sides. Equation (16) gives us

$$\phi^{**} = \frac{\alpha}{U_H A}; \qquad 1-\phi^{**} = \frac{\beta}{U_L A}.$$

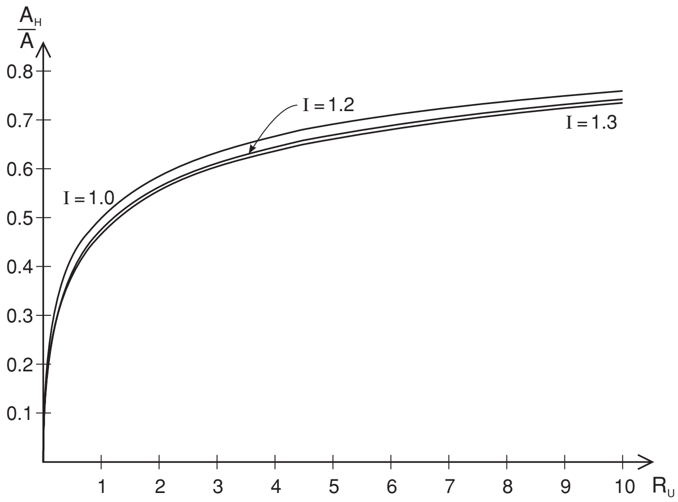

Figure 2. The effect of the distribution of the heat exchanger area $\frac{A_H}{A}$ versus R_u, if the total area is fixed (equation (25)) for values of $I = 1$, 1.2 and 1.3.

Thus, dimensionless power $p^{**} = \frac{P}{AU_H T_H}$ is given by

$$p^{**} = \frac{(1 - \sqrt{I\mu})^2}{\frac{1}{\phi^{**}} + \frac{I}{(1-\phi^{**})R_u}},$$

where $R_u = \frac{U_L}{U_H}$. In optimizing the power with respect to ϕ^{**}, we have

$$\phi^{**}_{mp} = \frac{\sqrt{R_u}}{\sqrt{I} + \sqrt{R_u}} \tag{24}$$

or equivalently

$$\frac{\beta}{\alpha} = \sqrt{I}\sqrt{R_u}; \qquad \frac{A_L}{A_H} = \sqrt{\frac{I}{R_u}} = \sqrt{I\frac{U_H}{U_L}}.$$

Then, the optimal distribution of the heat exchangers areas is

$$A_H = \frac{A}{1 + \sqrt{I\frac{U_H}{U_L}}} \tag{25}$$

$$A_L = \frac{A}{1 + \sqrt{\frac{U_L}{IU_H}}} \tag{26}$$

which has been reported in [9, 32], if $I = 1$. Moreover, this coincides with that of [36], if there is no heat leak in their model. However, [32, 36] have used another thermoeconomic optimization criterion.

The optimal distribution of the heat exchanger area $\frac{A_H}{A}$ with respect to $R_u\left(\frac{U_L}{U_H}\right)$ is shown in figure 2. This figure shows that a larger fraction of the area supply should be allocated to the heat exchanger whose overall heat transfer coefficient is lower ($I = 1$) [9] . In general, if R_u decreases, because the effectiveness of the hot side heat exchanging is proportionally higher, the hot side area always decreases, but quickly for $R_u < 1$. This behavior is the same if the internal dissipation increases ($I > 1$). One analogous interpretation corresponds to R_u increased. Also, the optimal distribution of the heat exchanger area $\frac{A_L}{A}$ with respect to R_u is included in figure 2; since $\frac{A_L}{A}$ is clearly the complement of $\frac{A_H}{A}$.

4. Discussions and conclusions

After the CA-engine, a new approach to thermodynamics was proposed: finite time thermodynamics. This approach took into account time in the analysis of thermodynamic processes and emphasized maximum power as an interesting bound. Inspired by this approach, Rubin [12] used it to study the endoreversible engine, which he defined as: *an engine such that during its operation its working fluid undergoes reversible thermodynamics.* In many applications, the power plants are more convenient to use than the CA-engine. While heat exchanges are sequential processes in the CA-engine, they are simultaneous processes in the power plants. The differences arisen in relation to this have been considered previously in [37].

Now if in the Carnot-like power plant model herein presented we consider additionally that the times of the two isothermal processes (the CA-like model [4]) are, respectively, t_H and t_L, the connecting adiabatic branches are often assumed to proceed in negligible time [6], and if the cycle contact total time t is [12]

$$t = t_H + t_L, \tag{27}$$

then, we have the following corollary.

Corollary 1. *For* $\phi^t_{mp} = \frac{t_H}{t}$ *the partial criterion is satisfied.*

For maximum power, we have found the following: if we consider the Carnot-like power plant model (steady-state), then ϕ_{mp} satisfies equation (20); but if not, for the CA-like model ϕ_{mp} is given by [1]

$$\phi^t_{mp} = \frac{1}{\sqrt[3]{I}+1} = \phi^t_{me} = \phi^t_{mec} = \phi^t_{pme} = \phi^t_{m\Omega}. \tag{28}$$

The values of ϕ_{mp} are different because the last equation was obtained including simultaneously the constraint of cycle contact total time (equation (27)) together with the constraint given by equation (11). This case is developed with further details in [1, 22]. For instance, equations (25) and (26) corresponding to the optimal distribution of the heat exchangers areas (CA-like model) change to

$$A^t_H = \frac{A}{1+\sqrt[3]{I}\sqrt{\frac{U_H}{U_L}}} \qquad \text{and} \qquad A^t_L = \frac{A}{1+\frac{1}{\sqrt[3]{I}}\sqrt{\frac{U_L}{U_H}}}.$$

An outstanding conclusion for the maximum ecological function for the Carnot-like power plant model herein presented (or CA-like model) is the following total criterion.

Criterion 2. *If there is no leak in the Carnot-like power plant model (or CA-like model), then the points in which the function E achieves a maximum value* ($x_{mE}, \phi_{mE}, \phi^*_{mE}, \phi^{**}_{mE}$ *or* $x_{mE}, \phi^t_{mE}, \phi^{t*}_{mE}, \phi^{t**}_{mE}$) *are*

$$x_{mE} = \sqrt{\frac{\mu(1+\mu)}{2I}},$$

$$\phi_{mE} = \frac{1}{\sqrt{I}+1}\left(\text{or } \phi^t_{mE} = \frac{1}{\sqrt[3]{I}+1}\right)$$

$$\phi^*_{mE} = \frac{1}{1+\sqrt{cI}} \left(\text{or } \phi^{t*}_{mE} = \frac{1}{1+\sqrt{c}\sqrt[3]{I}}\right)$$

$$\phi^{**}_{mE} = \frac{\sqrt{R_u}}{\sqrt{I}+\sqrt{R_u}} \left(\text{or } \phi^{t**}_{mE} = \frac{\sqrt{R_u}}{\sqrt[3]{I}+\sqrt{R_u}}\right)$$

and the efficiency at maximum ecological function can be expressed approximately as the semi-sum of the Carnot-like efficiency and the efficiency at maximum power (semi-sum property) of the Carnot-like power plant model (or CA-like model). This maximization is valid for any law of heat transfer that satisfies some of the following constraints:

$$\alpha + \beta = \gamma \quad \text{or} \quad a\alpha + b\beta = C \tag{29}$$

$$A_H + A_L = A \tag{30}$$

$$t_H + t_L = t, \tag{31}$$

where γ, C, A *and* t *are constants.*

Indeed, if there is no heat leak, the maximum ecological function [16] and [17] (which coincides with the $\dot{\Omega}$ criterion [30]) satisfies the semi-sum property independently from the heat transfer law used; i.e. if x_{mE} is the point in which the function E achieves a maximum value, this point is always given by [1]

$$x_{mE} = \sqrt{\frac{\mu(1+\mu)}{2I}},$$

equations (20), (22), (24), (28), by the partial criterion herein presented, and equations (28) and

$$\phi^{t*}_{mE} = \frac{1}{1+\sqrt{c}\sqrt[3]{I}} \qquad \text{and} \qquad \phi^{t**}_{mE} = \frac{\sqrt{R_u}}{\sqrt[3]{I}+\sqrt{R_u}},$$

which were found in [1], are also valid for any law of heat transfer with internal thermal conductances, overall heat transfer coefficients, areas and times constrained by one or two of the equations (29), (30) or (31).

However, the remarkable conclusion of this work is that it can find the maximum power, for the Carnot-like power plant model (or CA-like model) herein presented, by

$$\left.\frac{\partial P}{\partial \phi}\right|_{\phi_{mp}} = 0 \qquad \text{and} \qquad \left.\frac{\partial^2 P}{\partial \phi^2}\right|_{\phi_{mp}} < 0,$$

where ϕ corresponds to one of the variables that satisfies some of the following constraints (29), (30) or (31). The optimal values obtained given by

$$\phi_{op} = \frac{1}{\sqrt{I}+1} \left(\text{or } \phi^t_{op} = \frac{1}{\sqrt[3]{I}+1}\right)$$

$$\phi^*_{op} = \frac{1}{1+\sqrt{cI}} \left(\text{or } \phi^{t*}_{op} = \frac{1}{1+\sqrt{c}\sqrt[3]{I}}\right)$$

$$\phi^{**}_{op} = \frac{\sqrt{R_u}}{\sqrt{I}+\sqrt{R_u}} \left(\text{or } \phi^{t**}_{op} = \frac{\sqrt{R_u}}{\sqrt[3]{I}+\sqrt{R_u}}\right)$$

are fundamental because they are the same for the other operation regimes of the model (efficiency, ecological function, efficient power or $\dot{\Omega}$ criterion) and are independent from the heat transfer law used.

Also it is outstanding that the parameter x (the internal isentropic temperature ratio) can be considered as the fundamental characteristic parameter of these models. This is the only parameter that changes its optimal value according to the engine operation conditions. The remaining parameters ϕ, ϕ^*, ϕ^{**} maintain their optimal value independently of the operation condition of the power plant.

Finally, the methodology and the partial criterion can be applied to other objective functions which are algebraic linear combinations of the objective functions herein presented, for instance $P + \eta$, and can be extended to other models of irreversible engines [19–21]. Further work is underway.

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References

[1] Aragón-González G, Canales-Palma A, León-Galicia A and Morales-Gómez J R 2008 Maximum power, ecological function and efficiency of an irreversible Carnot cycle. A cost and effectiveness optimization *Braz. J. Phys.* **38** 543–50
[2] Chen L, Zhou J, Sun F and Wu C 2004 Ecological optimization for generalized irreversible Carnot engines *Appl. Energy* **77** 327–38
[3] Sanchez Salas N, Velasco S and Calvo Hernández A 2002 Unified working regime of irreversible Carnot-like heat engines with nonlinear heat transfer laws *Energy Convers. Manage.* **43** 2341–8
[4] Chen J 1994 The maximum power output and maximum efficiency of an irreversible Carnot heat engine *J. Phys. D: Appl. Phys.* **27** 1144–9
[5] Yan Z and Chen L 1995 The fundamental optimal relation and the bounds of power output and efficiency for an irreversible Carnot engine *J. Phys. A: Math. Gen.* **28** 6167–75
[6] Gordon J M 1991 On optimizing maximum power heat engines *J. Appl. Phys.* **69** 1–7
[7] Bejan A 1982 *Entropy Generation Through Heat and Fluid Flow* (New York: Wiley)
[8] Bejan A 1988 *Advanced Engineering Thermodynamics* (New York: Wiley)
[9] Bejan A 1995 Theory of heat transfer-irreversible power plants: II. The optimal allocation of heat exchange equipment *Int. J. Heat Mass Transfer* **38** 433–44
[10] Bejan A 1996 *Entropy Generation Minimization* (Boca Raton, FL: CRC Press)
[11] Curzon F L and Ahlborn B 1975 Efficiency of a Carnot engine at maximum power output *Am. J. Phys.* **43** 22–4
[12] Rubin M H 1979 Optimal configurations of a class of irreversible heat engines *Phys. Rev.* A **15** 2094–102
[13] Aragón-González G, Canales-Palma A, León-Galicia A and Musharrafie-Martínez M 2003 A criterion to maximize the irreversible efficiency in heat engines *J. Phys. D: Appl. Phys.* **36** 280–7
[14] Angulo-Brown F 1991 An ecological optimization–criterion for finite-time heat engines *J. Appl. Phys.* **69** 7465–9
See also comment by Yan Z J 1993 *Appl. Phys.* **73** 3583
[15] Angulo-Brown F, Arias-Hernández L A and Páez-Hernández R 1997 A general property of endoreversible thermal cycles *J. Appl. Phys.* **81** 2973–9
[16] Angulo-Brown F, Arias-Hernández L A and Páez-Hernández R 1999 A general property of non-endoreversible thermal cycles *J. Phys. D: Appl. Phys.* **32** 1415–20
See also comment by Yan Z J 2000 *Phys. D: Appl. Phys.* **33** 876
[17] Arias-Hernández L A, Ares de Parga G and Angulo-Brown F 2003 On some nonendoreversible engine models with nonlinear heat transfer laws *Open Syst. Inf. Dyn.* **10** 351–75
[18] Hoffman K H, Burzler J M and Shuberth S 1997 Endoreversible thermodynamics *J. Non-Equilib. Thermodyn.* **22** 311–55
[19] Durmayaz A, Sogut O S, Sahin B and Yavuz H 2004 Optimization of thermal systems based on finite time thermodynamics and thermoleconomics *Prog. Energy Combust. Sci.* **30** 175–217

[20] Chen L, Wu C and Sun F 1999 Finite time thermodynamic optimization or entropy generation minimization of energy systems *J. Non-Equilib. Thermodyn.* **24** 327–59

[21] Chen L and Sun F 2004 *Advances in Finite-time Thermodynamics: Analysis and Optimization* (New York: Nova Science Publishers)

[22] Aragón-González G, Canales-Palma A, León-Galicia A and Morales-Gómez J R 2006 Optimization of an irreversible Carnot engine in finite time and finite size *Rev. Mex. Fis.* **52** 309–14

[23] Lewins J D 2000 The endo-reversible thermal engine: cost and effectiveness optimization *Int. J. Mech. Eng. Educ.* **28** 41–6

[24] De Vos A 1985 Efficiency of some heat-engines at maximum-power conditions *Am. J. Phys.* **53** 570–3

[25] De Vos A 1987 Reflections on the power delivered by endoreversible engines *J. Phys. D: Appl. Phys.* **20** 232–6

[26] Chen L, Sun F and Wu C 1999 Effect of heat transfer law on the performance of a generalized irreversible Carnot engine *J. Phys. D: Appl. Phys.* **32** 99–105

[27] Chen L, Zhi X, Sun F and Wu A C 2004 Optimal configuration and performance for a generalized Carnot cycle assuming the heat-transfer generalized law *Open Syst. Inf. Dyn.* **9** 85–96

[28] Li J, Chen L and Sun F 2007 Optimal configuration of a class of endoreversible heat-engines for maximum power-output with linear phenomenological heat-transfer law *Appl. Energy* **84** 944–57

[29] Chen L, Li J and Sun F 2008 Generalized irreversible heat-engine experiencing a complex heat-transfer law *Appl. Energy* **85** 52–60

[30] Calvo A, Medina A, Roco J M, White J A and Velasco S 2001 Unified optimization criterion for energy converters *Phys. Rev.* E **63** 037102

[31] Yilmaz T 2006 A new performance criterion for heat engines: efficient power *J. Energy Inst.* **79** 38–41

[32] Sahin B and Kodal A 2001 Performance of an endoreversible heat engine based on a new thermoeconomic optimization criterion *Energy Convers. Manage.* **42** 1085–93

[33] Aragón-González G, Canales-Palma A and León-Galicia A 2000 Maximum irreversible work and efficiency in power cycles *J. Phys. D: Appl. Phys.* **33** 1403–10

[34] Arias-Hernández L A, Angulo-Brown F and Páez-Hernández R T 2008 First-order irreversible thermodynamic approach to a simple energy converter *Phys. Rev.* E **77** 011123

[35] Salah EL-Din M M 2001 Second law analysis heat engines with variable temperature heat reservoirs *Energy Convers. Manage.* **42** 189–200

[36] Sahin B and Kodal A 2003 Finite size thermoeconomic optimization for irreversible heat engines *Int. J. Ther. Sci.* **42** 777–82

[37] Wu C, Kiang R L, Lopardo V J and Karpouzian G N 1993 Finite-time thermodynamics and endoreversible heat engines *Int. J. Mech. Eng. Educ.* **21** 337–46